

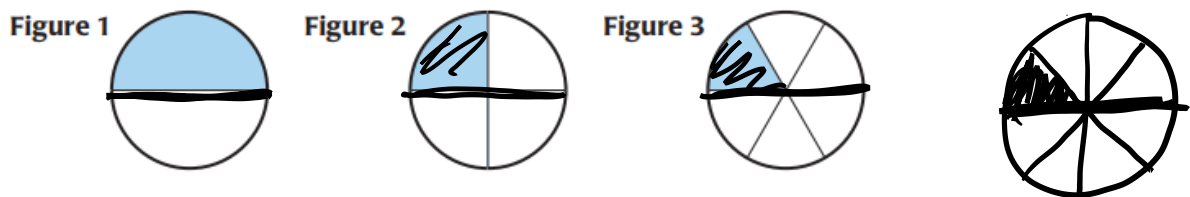
## 2.1 Patterns and Inductive Reasoning

### Vocabulary

- ❖ **Conjecture:** an unproven statement that is based on observations
- ❖ **Inductive reasoning:** a process of observing data, recognizing patterns, and making generalizations about those patterns
- ❖ **Counterexample:** a specific case in which the conjecture is false

### Describe a Visual Pattern

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.



Example 1:

Sketch the next figure in the pattern.



### Describe a number pattern

Describe the pattern in the numbers  $-7, -21, -63, -189, \dots$  and write the next three numbers in the pattern.

$$\begin{array}{ccc} \cup & \cup & \cup \\ \times 3 & \times 3 & \times 3 \end{array}$$

$$-567, -1701, -5103$$

## Example 2

Describe a pattern in the sequence of numbers. Predict the next number.

<p>2. 4, -20, 100, -500, ...</p> <p>    <math>\cup</math>    <math>\cup</math></p> <p>    <math>\times -5</math>  <math>\times -5</math></p> <p>                    2500</p>	<p>3. 10, 20, 40, 70, 110, ...</p> <p>    <math>\cup</math>    <math>\cup</math>    <math>\cup</math>    <math>\cup</math></p> <p>    +10  +20  +30  +40</p> <p>                    160</p>
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## Make a Conjecture

Complete the conjecture.

Conjecture: The sum of the first  $n$  even positive integers is     ?    .

**Solution** List some specific examples and look for a pattern.

Examples:

first even integer:

$$2 = 1(2)$$

sum of first two even positive integers:

$$2 + 4 = 6 = 2(3)$$

sum of first three even positive integers:

$$2 + 4 + 6 = 12 = 3(4)$$

sum of first four even positive integers:

$$2 + 4 + 6 + 8 = 20 = 4(5)$$

Conjecture: The sum of the first  $n$  even positive integers is

$$n(n+1)$$

## Finding a Counterexample

Show the conjecture is false by finding a counterexample.

Conjecture: If the difference of two numbers is odd, then the greater of the two numbers must also be odd.

$$10 - 9 = 1$$

$$10 - 5 = 5$$

$$10 - 7 = 3$$

### Example 3

Complete the conjecture based on the pattern you observe.

$$1 = 1$$

$$1 + 2 = 3 = \frac{2(2 + 1)}{2}$$

$$1 + 2 + 3 = 6 = \frac{3(3 + 1)}{2}$$

$$1 + 2 + 3 + 4 = 10 = \frac{4(4 + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 = 15 = \frac{5(5 + 1)}{2}$$

$$1 + 2 + 3 + 4 + 5 + 6 = 21 = \frac{6(6 + 1)}{2}$$

$$\frac{n(n+1)}{2}$$

Conjecture: The sum of the first  $n$  positive numbers is                     .

### Example 4

Show the conjecture is false by finding a counterexample.

$$-2 - (-4) = 2$$

Conjecture: The difference of two negative numbers is always negative.

$$-1 - (-3) = 2$$

Find the  $n^{\text{th}}$  term

Stage $n$	1	2	3	4	5
Outcome	2	6	10	14	18

What would the outcome be for Stage 6<sup>th</sup> 22 7<sup>th</sup> 26 8<sup>th</sup> 30  $4n-2$

Write an expression for the  $n^{\text{th}}$  Stage  $4n-2$

Fill in the 20<sup>th</sup> 78 65<sup>th</sup> 258

Stage	1	2	3	4	5
Outcome	3	8	13	18	23

What would the outcome be for Stage 6<sup>th</sup> 29 7<sup>th</sup> 33 8<sup>th</sup> 38  $5n$

Write an expression for the  $n^{\text{th}}$  Stage  $5n-2$

Fill in the 20<sup>th</sup> 98 53<sup>th</sup> 263

**Deductive Reasoning:** a process of showing certain statements follow logically from agreed upon assumptions and proven facts

**Example:**

Solve the equation for x. Give a reason for each step in the process.

$$3(2x - 5) + 20 = 5(2x - 5)$$

Steps	Reason